# CAMBRIDGE INTERNATIONAL EXAMINATIONS <br> General Certificate of Education Ordinary Level <br> MATHEMATICS (SYLLABUS D) <br> Paper 2 <br> May/June 2003 <br> 2 hours $\mathbf{3 0}$ minutes <br> Additional Materials: Answer Booklet/Paper <br> Electronic calculator Geometrical instruments Graph paper (1 sheet) Mathematical tables (optional) 

## READ THESE INSTRUCTIONS FIRST

Write your answers and working on the separate Answer Booklet/Paper provided.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

## Section A

Answer all questions.

## Section B

Answer any four questions.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 100.
You are expected to use an electronic calculator to evaluate explicit numerical expressions. You may use mathematical tables as well if necessary.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

## Section A [52 marks]

Answer all questions in this section.
1 (a) (i) Evaluate $\frac{4.8^{2}-1.7^{2}}{4.8 \times 1.7}$.
(ii) Find a value of $x$ for which $\sin x^{\circ}=\tan 12^{\circ}+\cos 46^{\circ}$.
(b) The diagram shows a framework $A B C D$.
$A D=2.2 \mathrm{~m}, B D=1.9 \mathrm{~m}$ and $B \hat{C} D=42^{\circ}$.
$A \hat{B} D=B \hat{D} C=90^{\circ}$.

Calculate

(i) $A \hat{D} B$,
(ii) $B C$.
(c) A vertical flagpole, 18 m high, stands on horizontal ground.

Calculate the angle of elevation of the top of the flagpole from a point, on the ground, 25 m from its base.

2 (a) Factorise completely $20 t^{2}-5$.
(b) Express as a single fraction in its simplest form

$$
\begin{equation*}
\frac{7}{2 x}-\frac{5}{3 x} \tag{2}
\end{equation*}
$$

(c) Tickets for a concert were priced at $\$ 5, \$ 8$ and $\$ 12$.

The number of $\$ 5$ tickets sold was twice the number of $\$ 8$ tickets.
The number of $\$ 12$ tickets sold was 80 more than the number of $\$ 8$ tickets.
The number of $\$ 8$ tickets sold was $x$.
(i) Find an expression, in terms of $x$, for the total sum of money received from the sale of the tickets.
(ii) Given that $\$ 9360$ was received from the sale of the tickets, form an equation in $x$. Solve this equation and hence find the total number of tickets that were sold.

3 In 2001 the price of one litre of petrol was 72 cents.
(a) $65 \%$ of this price is 'tax' and the remainder is 'other costs'.
(i) Find, in its simplest form, the ratio of tax to other costs.

Give your answer in the form $m: n$, where $m$ and $n$ are integers.
(ii) Calculate how much tax is paid on one litre of petrol.
(b) Maureen bought as many complete litres of petrol as she could with a $\$ 20$ note ( $\$ 1=100$ cents).
(i) Calculate how many litres she bought.
(ii) Calculate how much change she received.
(c) In 2002 the price of one litre of petrol was 81 cents.

Calculate the percentage increase in the price of petrol from 2001 to 2002.
(d) The price of petrol in 2001 was $10 \%$ less than the price in 2000.

Calculate the price of one litre of petrol in 2000.
(e) Andrew's car will travel 480 km on a full tank of petrol.

He starts a journey of 620 km with a tank which is half full. He wants to stop only once for petrol.
Between what distances from the start of his journey must he stop for petrol?

4

$B D$ is a diameter of the circle, centre $O$.
$C$ and $A$ are two points on the circle.
$A B$ and $D C$, when produced, meet at $E$.
$A \hat{O} B=110^{\circ}$ and $B \hat{D} C=23^{\circ}$.
(a) Find
(i) $A \hat{D} O$,
(ii) $B \hat{A} C$,
(iii) $C \hat{B} D$,
(iv) $C \hat{E} B$.
(b) $M$ is the midpoint of $C D$.
(i) Explain why triangle $O M D$ is similar to triangle $B C D$.
(ii) Write down the value of $\frac{\text { Area of } \triangle O M D}{\text { Area of } \triangle B C D}$.

5 (a) One hundred and sixty students took an examination.
The table shows the marks needed for each grade.
The cumulative frequency curve shows the distribution of their marks.

(i) Use the graph to estimate
(a) the median,
(b) the interquartile range,
(c) the number of students who were awarded a Grade C.
(ii) A pie chart was drawn to illustrate the grades awarded to the students.

Calculate the angle of the sector which represented the number of students who were awarded a Grade C.
(b) An ordinary unbiased die has faces numbered $1,2,3,4,5$ and 6 .

Sarah and Terry each threw this die once.
Expressing each answer as a fraction in its lowest terms, find the probability that
(i) Sarah threw a 7,
(ii) they both threw a 6 ,
(iii) neither threw an even number,
(iv) Sarah threw exactly four more than Terry.


The natural numbers $1,2,3, \ldots$ are written, in a clockwise direction, on a circular grid as shown in the diagram.
There are four numbers in each ring.
The numbers $1,2,3$, and 4 are in the first ring.
The numbers 5, 6, 7 and 8 are in the second ring.
The following numbers fill up the other rings in the same way.
(a) Write down the numbers in the fourth ring.
(b) Write down the largest number in the tenth ring.
(c) The sum, $S_{n}$, of the four numbers in the $n$th ring, where $n=1,2$ and 3 , is given in the table below.

| $n$ | 1 | 2 | 3 | 4 |
| :---: | ---: | ---: | ---: | ---: |
| $S_{n}$ | 10 | 26 | 42 |  |

(i) Write down the value of $S_{4}$.
(ii) Find, in its simplest form, an expression, in terms of $r$, for $S_{r}$.
(iii) In which ring is the sum of the four numbers equal to 1018 ?

## Section B [48 marks]

Answer four questions in this section.
Each question in this section carries 12 marks.

7 [The value of $\pi$ is 3.142 , correct to three decimal places.]
[The surface area of a sphere is $4 \pi r^{2}$.]
[The volume of a sphere is $\frac{4}{3} \pi r^{3}$.]
A closed container is made by joining together a cylinder of radius 9 cm and a hemisphere of radius 9 cm as shown in Diagram I.
The length of the cylinder is 18 cm .
The container rests on a horizontal surface and is exactly half full of water.


Diagram I
(a) Calculate the surface area of the inside of the container that is in contact with the water. Give your answer correct to the nearest square centimetre.
(b) Show that the volume of the water is $972 \pi \mathrm{~cm}^{3}$.
(c) The container is held with its axis vertical, the hemisphere being at the bottom, as shown in Diagram II.

Calculate the depth of the water.


Diagram II
(d) The container is now placed with its circular end on a horizontal surface as shown in Diagram III.
Find the depth of the water.


Diagram III

## 8 Answer the whole of this question on a sheet of graph paper.

Temperatures were recorded over a nine hour period.
The table below shows the temperature, $y^{\circ} \mathrm{C}$, at various times.

| Time $(x$ hours $)$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Temperature $\left(y^{\circ} \mathrm{C}\right)$ | 2 | -1 | -2 | -1.4 | 0 | 2 | 3.5 | 3.4 | 2.4 | 0.6 |

(a) Using a scale of 1 cm to represent 1 hour, draw a horizontal $x$-axis for $0 \leqslant x \leqslant 9$.

Using a scale of 2 cm to represent $1^{\circ} \mathrm{C}$, draw a vertical $y$-axis for $-2 \leqslant y \leqslant 4$.
On your axes, plot the points given in the table and join them with a smooth curve.
(b) Use your graph to find an estimate for
(i) the temperature when $x=5.5$,
(ii) the difference between the highest and lowest temperatures,
(iii) how long, in hours and minutes, the temperature was above $2^{\circ} \mathrm{C}$.
(c) (i) By drawing a tangent, find the gradient of the curve at the point where $x=8$.
(ii) State briefly what this gradient represents.
(d) The curve from $x=0$ to $x=2$ has the equation $y=x^{2}+B x+C$.

Find the value of $C$ and the value of $B$.

9


The diagram shows the position of a harbour, $H$, and three islands $A, B$ and $C$.
$C$ is due North of $H$.
The bearing of $A$ from $H$ is $062^{\circ}$ and $H \hat{A} B=128^{\circ}$.
$H A=54 \mathrm{~km}$ and $A B=31 \mathrm{~km}$.
(a) Calculate the distance $H B$.
(b) Find the bearing of $B$ from $A$.
(c) The bearing of $A$ from $C$ is $133^{\circ}$.

Calculate the distance $A C$.
(d) A lightship, $L$, is positioned due North of $H$ and equidistant from $A$ and $H$.

Calculate the distance $H L$.

10


Diagram I


Diagram II

Diagram I shows a quadrilateral, $A B C D$, in which $D A=A B=x$ centimetres and $B C=C D=y$ centimetres.
$A \hat{B} C=C \hat{D} A=90^{\circ}$.
(a) Show that the area of this quadrilateral is $x y$ square centimetres.
(b) Five of these quadrilaterals are joined together to make the shape shown in Diagram II.

The total area of this shape is $80 \mathrm{~cm}^{2}$.
(i) Show that the outside perimeter, $P$ centimetres, of this shape is given by

$$
\begin{equation*}
P=10 x+\frac{32}{x} . \tag{2}
\end{equation*}
$$

(ii) (a) In the case when $P=38$, show that $5 x^{2}-19 x+16=0$.
(b) Solve the equation $5 x^{2}-19 x+16=0$, giving both answers correct to two decimal places.
(c) Find the two possible values of $y$ when $P=38$.
(iii) (a) Calculate the value of $P$ when $x=y$.
(b) What is the special name given to the quadrilateral $A B C D$ when $x=y$ ?

11


The diagram shows triangles $A, B, C$ and $D$.
(a) Describe fully the single transformation which maps $A$ onto $B$.
(b) Find the matrix that represents the single transformation which maps $A$ onto $C$.
(c) $A$ is mapped onto $D$ by a clockwise rotation.

Find
(i) the angle of this rotation,
(ii) the coordinates of the centre of this rotation.
(d) The matrix $\left(\begin{array}{ll}2 & 0 \\ 0 & 1\end{array}\right)$ represents the transformation which maps triangle $A$ onto triangle $E$.
(i) Find the coordinates of the vertices of triangle $E$.
(ii) Describe fully the transformation that is represented by the matrix $\left(\begin{array}{ll}2 & 0 \\ 0 & 1\end{array}\right)$.
(iii) Find the matrix that represents the single transformation which maps triangle $E$ onto triangle $A$.

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